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The Poisson structure of a 4D spinning string

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Abstract. A model of a 4D open string with non-Grassmann spinning variables is considered. The nonlinear gauge, which is invariant under both Poincaré and scale transformations of spacetime, is used for subsequent studies. It is shown that the reduction of the canonical Poisson structure from the original phase space to the surface of constraints and gauge conditions gives the degenerated Poisson brackets. Moreover it is shown that such a reduction is non-unique. The concept of adjunct phase space is introduced. The consequences for subsequent relativistic invariant quantization are discussed. Deduced dependence of spin J from the square of mass μ^2 of the string generalizes the 'Regge spectrum' for conventional theory.

1. Introduction

The investigation of constrained dynamical systems started by Dirac [1] has continued in connection with gauge theory development. These studies have taken many directions; one being the modification of the conventional phase space concept (see, for example, [2]). In this paper, we suggest a new viewpoint on the phase space for some kinds of gauge systems and apply the suggested concept to an investigation of 4D string dynamics.

We start our study with the following simple example. Let the space $\mathcal{H}_N$ be the phase space of any dynamical system with N degrees of freedom; any point $M \in \mathcal{H}_N$ has the coordinates $p_1, q_1, \ldots, p_N, q_N$ which diagonalize the standard non-degenerated Poisson brackets: $\{p_i, q_j\} = \delta_{ij}$, $\{p_i, p_j\} = \{q_i, q_j\} = 0$. Let us consider the subset $V \subset \mathcal{H}_N$: $V = \{M \in \mathcal{H}_N \mid p_1 = 0,\ q_1 > 0\}$. What is the Poisson structure of the set V? It is clear that such a structure must be degenerated because $\operatorname{codim} V = 1$. The simplest foliation of the set V will be the following:

$$V = \cup_{c>0} V_c^0$$

where set $V_c^0 = \{M \in \mathcal{H}_N | p_1 = 0,\ q_1 = c,\ (c = \text{constant})\}$. It is well known that the 'correct' brackets for any set V_c^0 will be the Dirac brackets $\{\cdot,\cdot\}_1$ for the pair of (second type) constraints $p_1 = 0, q_1 - c = 0$. This bracket structure can be naturally extended on set V; the function $f_0 \equiv q_1$ will be an annihilator. The interesting fact is that the constructed brackets are non-unique. Indeed, we can introduce the other foliation of the set V: $V = \cup_{c>0} V_c^f$, where the subsets $V_c^f = \{M \in \mathcal{H}_N | p_1 = 0,\ f(q_1; q_2, p_2, \ldots) = c,\ (c = \text{constant})\}$ were defined with the help of some appropriate function f such that condition $0 < \frac{\partial f}{\partial q_1} < \infty$ holds. It is clear that the corresponding Dirac brackets $\{\cdot,\cdot\}_f$ differ from the brackets $\{\cdot,\cdot\}_1$; the annihilator for the new brackets is the function f. Thus, the reduction of the same Poisson structure from the original phase space $\mathcal{H}_N$ on some subset $V \subset \mathcal{H}_N$ can be ambiguous if the reduced brackets are degenerate. In general, the situation is the same if we consider the system of the first type

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of constraints $f_1, \ldots, f_k$, where $k < N$, instead of the single one $p_1 = 0$. Of course, this example is a special case of the general theory of degenerated Poisson brackets [3]. It was discussed in detail because the goal of our subsequent studies is to investigate this effect in string theory.

A satisfactory version of the 4D quantum (super)string has been the aim of the theoretical studies of many authors (see, for example, [4–8]). Moreover, many authors have constructed theories in arbitrary (non-critical) spacetime dimensions [9–11]. Of course, this list is incomplete: a detailed review is impossible here. Our suggested approach differs, to our knowledge, from others because it is founded on a new concept of adjunct phase space.

We consider the following model here. Let the fields $X_\mu(\xi^0, \xi^1)$ and $\Psi^A_\pm(\xi^0, \xi^1)$ interact with two-dimensional gravity $h_{ij}(\xi^0, \xi^1)$, where $\xi^1 \in [0, \pi]$ and $\xi^0 \in (-\infty, \infty)$, such that the dynamics is defined by the action constructed in accordance with the well-investigated manner [12]:

$$S = -\frac{1}{4\pi\alpha'} \int \mathrm{d}\xi^0\,\mathrm{d}\xi^1 \sqrt{-h}\{h^{ij}\partial_i X^\mu \partial_j X_\mu - \mathrm{i}\Theta e^j_I (\Gamma^0)_{AB} \overline{\Psi}^A \gamma_j \nabla^I \Psi^B\}. \quad (1)$$

The notations are the following: $h = \det(h^{ij})$, the vectors $e^j_I(\xi^0, \xi^1)$ are the vectors of 2D basis such that the equalities $h^{ij} = e^{iI} e^j_I$ take place and the matrices Γ^μ and γ_i are the Dirac matrices in the 4D and 2D spacetime respectively. The field $\boldsymbol{X} = X_\mu \boldsymbol{t}^\mu$ is the vector field in ('isotopical') Minkowski spacetime $E_{1,3}$; the fields Ψ^A with components $\Psi^A_\pm$ are the spinor fields in 2D space; index A is the spinor index in the space $E_{1,3}$ such that the fields $\Psi_\pm$ are the Majorana spinor fields in 4D spacetime. The numbers $\Psi^A_\pm$ are complex numbers, so there are no classical Grassmann variables in our action. The consideration of the spinning string without the Grassmann variables is not new (see, for example, [13]). In our opinion such an approach is justified here because the new fundamental variables will be complicated functions from the original fields X and Ψ.

To fix the gauge arbitrariness we demand, as usual, $e^j_I = \delta^j_I$ so that $h_{ij} = \mathrm{diag}(1, -1)$ and the equations of motion can be written in their simplest form. For fields X and Ψ we have $\partial_-\partial_+ X_\mu = 0$, $\partial_\mp \Psi_\pm = 0$. The equations of motion $\delta S/\delta h^{ij} = 0$ for gravity h lead, as is well known, to the equalities

$$F_{1\pm}(\xi) \equiv (\partial_\pm X)^2 \pm \frac{\mathrm{i}\Theta}{2} \overline{\Psi}_\pm \partial_\pm \Psi_\pm = 0 \quad (2)$$

where $\partial_\pm$ are derivatives with respect to cone parameters $\xi_\pm = \xi^1 \pm \xi^0$. The remaining gauge freedom [12]

$$\xi_\pm \longrightarrow \tilde{\xi}_\pm = \pm A(\pm\xi_\pm) \quad (3)$$

must be fixed by means of additional conditions. For our subsequent consideration it is important that the function $A(\xi)$ must satisfy the property

$$A(\xi + 2\pi) = A(\xi) + 2\pi \qquad A' \neq 0$$

in accordance with the standard boundary conditions for original variables X and Ψ: $X'_\mu(\xi^0, 0) = X'_\mu(\xi^0, \pi) = 0$, $\Psi_+(\xi^0, 0) = \Psi_-(\xi^0, 0)$ and $\Psi_+(\xi^0, \pi) = \epsilon\Psi_-(\xi^0, \pi)$, where $\epsilon = \pm$.

The original phase space $\mathcal{H}$ has the coordinates $\dot{X}_\mu \equiv \partial_0 X_\mu$, X_μ, ${\Psi^{+A}}_\pm$ and $\Psi^A_\pm$. As usual, the canonical Poisson bracket structure is the following:

$$\{\dot{X}_\mu(\xi), X_\nu(\eta)\} = -4\pi\alpha' g_{\mu\nu}\delta(\xi - \eta)$$

$$\{{\Psi^{A^+}}_\pm(\xi), \Psi^B_\pm(\eta)\} = \frac{8\pi\mathrm{i}\alpha'}{\Theta}(\Gamma^0)^{AB}\delta(\xi - \eta).$$

2. The additional gauge conditions and the adjunct phase space

The spinor variables give additional possibilities for constructing natural Poincaré-invariant structures on the (ξ^0, ξ^1)-plane†. For example, we can construct the following two-tensor:

$$\Omega_{ij}(\xi^0, \xi^1) = \tfrac{1}{2}(h_{im}h_{jn} + h_{in}h_{jm} - h_{ij}h_{mn})(\Gamma^0\Gamma^\mu)_{AB}\overline{\Psi}^A\gamma^m\Psi^B\partial^n X_\mu .$$

Other objects can be constructed too. Detailed investigations of these structures and the geometrical properties of the 'extended' world-sheet $(\xi^0, \xi^1) \to (X_\mu(\xi^0, \xi^1), \Psi^A_\pm(\xi_\pm))$ in some complex space would probably be interesting, but lie outside the framework of this paper. We include in our subsequent studies the string configurations $(\boldsymbol{X}, \Psi)$ which give the positive-defined quadratic form $\Omega_{ij}\, \mathrm{d}\xi^i\, \mathrm{d}\xi^j$ only. This demand means that two inequalities

$$\pm\overline{\Psi}_\pm\Gamma^\mu\Psi_\pm\partial_\pm X_\mu > 0 \tag{4}$$

hold for any point (ξ^0, ξ^1). To destroy the gauge freedom (3) we select the string configurations $(\boldsymbol{X}, \Psi)$ such that the conditions

$$F_{2\pm}(\xi) \equiv \overline{\Psi}_\pm\Gamma^\mu\Psi_\pm\partial_\pm X_\mu = \pm\frac{\kappa^2}{2} \tag{5}$$

hold for any non-zero constant $\kappa = \kappa[X, \Psi]$. Note that the equalities (5) are invariant both under Poincaré and scale transformations of the spacetime $E_{1,3}$, so we assume that the resulting theory will be attractive. Such invariance is the first motivation for the conditions (5). A second is that the gauge (5) naturally generalizes the well known light-cone gauge in a string theory. We discuss this in detail at the end of section 4.

It should be stressed that the restriction

$$\kappa[X, \Psi] = q \tag{6}$$

where q is some fixed input parameter is not suitable for a complete theory. Indeed, the different values of the constant κ correspond to different orbits of the gauge transformations (3), so that κ is a Teichmüller-like parameter. Consequently, the 'strong' restriction (6) will not be grounded because the gauge transformations (3) were forbidden by the 'weak' conditions (5) (discussion of this situation for general gauge systems can be found in [14]).

Note that the gauge (5) does not forbid the transformations (3) such that $A(\xi) \equiv \xi + c$, where c = constant. Obviously, they give the shifts $\xi^0 \to \xi^0 + c$, which correspond to dynamics.

We are going to study the Poisson structure of the set $\boldsymbol{V}$ of string configurations $(\boldsymbol{X}(\xi^0, \xi^1), \Psi(\xi_\pm))$ which are selected by the constraints (2) and 'weak' gauge conditions (5). It is a non-trivial problem, because the variation $\delta(\kappa[X, \Psi])$ is not defined by the variations of the coordinates of original phase space. Let us introduce the auxiliary minimal subspace $\mathcal{H}_1$ such that, first, the inclusion $\boldsymbol{V} \subset \mathcal{H}_1 \subset \mathcal{H}$ holds and, secondly, the Poisson structure on $\mathcal{H}_1$ is reduced unambiguously from the original phase space $\mathcal{H}$. Such a subspace is given by the equalities

$$F_i^{(n)} = 0 \qquad n \neq 0 \qquad i = 1, 2. \tag{7}$$

The constants $F_i^{(n)}$ are Fourier modes of 2π-periodic functions

$$F_i(\xi) = \begin{cases} F_{i+}(\xi) & \xi \in [0, \pi) \\ F_{i-}(-\xi) & \xi \in [-\pi, 0) \end{cases}$$

† Not only the first and second quadratic form of the world-sheet is used, as in the case of bosonic string.

which are well defined in accordance with the boundary conditions for the variables X and Ψ. The canonical Poisson structure on original phase space $\mathcal{H}$ gives the following brackets:

$$\{F_1^{*(n)}, F_2^{(n)}\} = 8\pi \mathrm{i}\alpha' n F_2^{(0)}.$$

Because $F_2^{(0)} = \kappa^2/2 > 0$ in our theory, the system (7) will be a second-type system of constraints so that natural brackets on space $\mathcal{H}_1$ will be corresponding Dirac brackets $\{\cdot,\cdot\}_1$. The condition $F_1^{(0)} = 0$ gives the reduction on set $\boldsymbol{V}$; obviously, $\mathrm{codim}\boldsymbol{V} = 1$. Analogously with the example, considered in the introduction, we can select the various foliations

$$\boldsymbol{V} = \bigcup_{q^2>0} \boldsymbol{V}_q^f \tag{8}$$

where the sets $\boldsymbol{V}_q^f \subset \boldsymbol{V}$ can be defined both by the restriction (6) and any more complicated conditions. As for the finite-dimensional case, any foliation (8) gives the Poisson structure on the set $\boldsymbol{V}$, which will be degenerate. Thus, the natural canonical structure of the original phase space $\mathcal{H}$ does not have the unique reduction to the set $\boldsymbol{V}$.

At first it seems that such indeterminacy can be ignored at the subsequent quantization. Indeed, we can quantize the brackets $\{\cdot,\cdot\}_1$ and construct the corresponding Fock space $\boldsymbol{H}_1$. After that we must select the physical vectors $|\psi\rangle \in \boldsymbol{H}_1$ which will be the solutions of the 'Shrödinger equation' $F_1^{(0)}|\psi\rangle = 0$ [1]. In our opinion, the ambiguity in determination of the Poisson structure of the manifold $\boldsymbol{V}$, which consists of all the physical information, leads to additional possibilities for quantization. Indeed, let any space $\mathcal{H}^{\mathrm{ad}}$ be any Poisson manifold with the Poisson brackets $\{\cdot,\cdot\}^0$. Suppose that the finite number of some constraints $\Phi_i(\ldots) = 0$, $i = 1,\ldots,l$ give the first-type system of constraints:

$$\{\Phi_i, \Phi_j\}^0 = C_{ijk}\Phi_k.$$

Suppose, that for the surface of these constraints $\boldsymbol{W} \subset \mathcal{H}^{\mathrm{ad}}$: $\boldsymbol{W} = \{M \in \mathcal{H}^{\mathrm{ad}} | \Phi_i = 0,\ i = 1,\ldots,l.\}$ the diffeomorphism $\boldsymbol{V} \approx \boldsymbol{W}$ takes place†. Thus, we have the following diagram:

$$\mathcal{H}_1 \supset \boldsymbol{V} \approx \boldsymbol{W} \subset \mathcal{H}^{\mathrm{ad}}. \tag{9}$$

We call any such space $\mathcal{H}^{\mathrm{ad}}$ the *adjunct phase space*. From the classical viewpoint, the manifold $\boldsymbol{W}$ is tantamount to the manifold $\boldsymbol{V}$, because it has the same information about the physical degrees of the freedom. As discussed above, the single-defined Poisson structure is absent both on the manifold $\boldsymbol{V}$ and the manifold $\boldsymbol{W}$, which is why there is no reason to prefer one to the other. Consequently, we can fulfil the subsequent quantization of the theory in terms of the space $\mathcal{H}^{\mathrm{ad}}$. It should be stressed that $\mathcal{H}_1 \not\approx \mathcal{H}^{\mathrm{ad}}$ in general as the Poisson manifolds, even for case $l = 1$. This means that the results here can differ from the conventional ones. The simplest example of adjunct phase space will be the manifold $D * \mathcal{H}_1$, where symbol $D*$ means any (not only Poisson) diffeomorphism. It is clear from the physical point of view. Indeed, if we have both physical and non-physical coordinates in some phase space, there is no reason to consider only Poisson-conserved transformations of such space. The suggested definition of the space $\mathcal{H}^{\mathrm{ad}}$ is more general, of course.

The main goal of this article is to construct the physically appropriate adjunct phase space for the dynamical system (1) and discuss the quantization. In our opinion, there are several reasons to refuse the description of string dynamics in terms of the space $\mathcal{H}_1$ (or the original phase space $\mathcal{H}$—it is equivalent). First, the standard approach leads to the additional dimensions for spacetime while the existence of such dimensions is not proven experimentally. Secondly, the conventional approaches (see, for example, [12]) lead, as is well known, to the linear Regge trajectories for free strings such that the slope α' is input parameter in a theory.

† This diffeomorphism must be conserved in the dynamics.

But the trajectories $s = \alpha'\mu^2 + c$, where the value $\alpha' \simeq 0.9\ \text{Gev}^{-2}$ is the universal constant, describe the spectrum of real particles well but only approximately. Indeed, the linearity means that the width of any resonance is equal to zero; the universality of the slope α' is connected with the absence of exotic particles [15]. We also now have stable experimental data on hadronic exotics [16], some of which give direct information about Regge trajectories with slopes $\alpha_g \neq 0.9\ \text{Gev}^{-2}$ [17]. As regards the form of the trajectory, the linear dependence gives a good approximation for light-flavoured mesons and baryons only (see, for example [18]). Moreover, the width of any real resonance does not equal zero, of course.

As can be seen, the construction of a 4D free string model, while taking into account the small nonlinearity of the trajectories and the existence of the different slopes, can be very interesting.

3. The world-sheet geometry

We will define next the adjunct phase space $\mathcal{H}^{\text{ad}}$, the subset $\boldsymbol{W}$ and construct the corresponding diffeomorphism $\boldsymbol{V} \approx \boldsymbol{W}$ in accordance with diagram (9). The coordinates in the space $\mathcal{H}^{\text{ad}}$, which will be introduced in the following section, naturally fall into two groups: the finite number of 'external' variables which are transformed as the tensors under the Lorentz transformations of the spacetime $E_{1,3}$ and some 'internal' scalar variables. In order to define these quantities, we consider in this section the geometrical construction which is quite natural for the studied model. Some parts of the section will be analogous to corresponding parts in [19, 20], so we give some formulae without detailed proof.

Let the constant κ be the constant existing for given fields X_μ and $\Psi_\pm$ in accordance with the conditions (5). We introduce the tensors

$$L^\mu_\pm = \frac{1}{\kappa}\overline{\Psi}_\pm\Gamma^\mu\Psi_\pm \qquad G^{\mu\nu}_\pm = \frac{\mathrm{i}}{2\kappa}\overline{\Psi}_\pm(\Gamma^\mu\Gamma^\nu - \Gamma^\nu\Gamma^\mu)\Psi_\pm$$

which carry the full information about Majorana spinors $\Psi_\pm$ and satisfy the properties $L_\mu G^{\mu\nu} = 0$, $L^\mu L^\nu = G^\mu_\rho G^{\rho\nu}$. After that we define the pair of vectors $N^\mu_\pm$:

$$N^\mu_\pm = \frac{1}{\kappa}\partial_\pm X^\mu + \frac{\mathrm{i}\Theta}{2\kappa^2}\overline{\Psi}_\pm\partial_\pm\Psi_\pm L^\mu_\pm.$$

According to the equalities (2) and (5) these vectors are light-like and satisfy the conditions

$$L^\mu_\pm N_{\mu\pm} = \pm\tfrac{1}{2}. \tag{10}$$

Let us define eight vectors $e_{\mu\pm}$ $\mu = 0, \ldots, 3$.

$$(e_{0\pm})^\mu = L^\mu_\pm \pm N^\mu_\pm \qquad (e_{3\pm})^\mu = \pm L^\mu_\pm - N^\mu_\pm$$
$$(e_{1\pm})^\mu = \mp 2G^{\mu\nu}_\pm N_{\pm\nu} \qquad (e_2)^\mu = \varepsilon^{\mu\nu\lambda\rho}(e_3)_\nu(e_0)_\lambda(e_1)_\rho.$$

The direct verification allows us to state that these vectors give the pair of the orthonormal bases in space $E_{1,3}$ for every point (ξ^0, ξ^1) of the world-sheet. Furthermore, it is more convenient to deal with the vector-matrices

$$\boldsymbol{E}_\pm = e^\mu_\pm\sigma_\mu \qquad \sigma_0 = 1$$

than the bases $e_{\mu\pm}$. So, we define $SL(2, C)$-valued chiral field $K = K(\xi^0, \xi^1)$ by means of the formula

$$\boldsymbol{E}_- = K\boldsymbol{E}_+K^+. \tag{11}$$

The free equations of motion for the original fields X_μ and $\Psi_\pm$ lead to the 'conservation laws' $\partial_\pm \boldsymbol{E}_\mp = 0$. Consequently, we have the equation for chiral field $K = K(\xi^0, \xi^1)$:

$$\partial_-(K^{-1}\partial_+K) = 0. \tag{12}$$

There is a special case for the well known Wess–Zumino–Novikov–Witten equation [21, 22]. The left and right currents

$$Q_- = -(\partial_- K)K^{-1} \qquad Q_+ = K^{-1}(\partial_+ K)$$

for the defined chiral field K satisfy the equations $\partial_\mp Q_\pm = 0$. As can be proven with the help of the boundary conditions for the original string variables $\boldsymbol{X}$ and Ψ, the $sl(2, C)$-valued function

$$Q(\xi) = \begin{cases} Q_+(\xi) & \xi \in [0, \pi] \\ -\sigma_1 Q_-(-\xi)\sigma_1 & \xi \in [-\pi, 0] \end{cases}$$

is continuous and can be extended 2π-periodically and continuously throughout the real axis.

Let us consider the following auxiliary linear system with 2π-periodic coefficients:

$$T'(\xi) + Q(\xi)T(\xi) = 0. \tag{13}$$

This system plays a central role in our subsequent considerations. This role is conditioned by the possibility of reconstruction of the original string variables $\partial_\pm X_\mu$ and $\Psi_\pm$ through the matrix solution $T(\xi)$ of the system (13).

Indeed, the chiral field $K(\xi^0, \xi^1)$ can be written in the form [22]:

$$K(\xi^0, \xi^1) = T_-(\xi_-)T_+^{-1}(\xi_+) \tag{14}$$

for some matrices $T_\pm \in \mathrm{SL}(2, C)$. Using the definition of the field K, we have the formulae

$$\boldsymbol{E}_\pm(\xi_\pm) = \mathrm{T}_\pm(\xi_\pm)\boldsymbol{E}_0\mathrm{T}_\pm^\dagger(\xi_\pm) \tag{15}$$

where $\boldsymbol{E}_0 = \boldsymbol{t}^\mu\sigma_\mu$ is the matrix representation of the stationary basis $\boldsymbol{t}^\mu$. In accordance with the definition of matrix Q, we have the equalities

$$T_+(\xi_+) = T(\xi_+) \qquad T_-(\xi_-) = \mathrm{i}\sigma_1 T(-\xi_-)$$

where $T(\xi)$ is matrix-solution of the system (13) such that condition $\det T = 1$ holds. So, we can reconstruct the vector matrices $\boldsymbol{E}_\pm(\xi_\pm)$ through the matrix $T(\xi)$. If the constant κ is given, we can reconstruct the original string variables too. Thus the following one-to-one correspondence takes place:

$$\boldsymbol{V}/E_{1,3} \approx (T(\xi), \kappa) \tag{16}$$

where as $E_{1,3}$ we denote the group of the translations $\boldsymbol{X} \to \boldsymbol{X} + A$. The evident formulae for the reconstruction of the original string variables can be deduced analogously just as in [20]†. For example, we have for the matrices $\partial_\pm\hat{X}(\xi_\pm) \equiv \partial_\pm X^\mu\sigma_\mu$:

$$\partial_\pm\hat{X}(\xi_\pm) = \pm T^+(\pm\xi_\pm)R(\pm\xi_\pm)T(\pm\xi_\pm) \tag{17}$$

where the matrix $R(\xi) = \mathrm{diag}(\kappa, -2\Theta\mathrm{Re}Q_{21}(\xi))$. If we select the Weyl representation for Γ-matrices, the explicit expressions for reconstructed spinors are quite simple:

$$\Psi_\pm = \sqrt{\kappa}\begin{pmatrix} \varphi_\pm \\ -\sigma_2\varphi_\pm^* \end{pmatrix} \tag{18}$$

where $\varphi_\pm(\xi_\pm) = \binom{t_{21}(\pm\xi_\pm)}{t_{22}(\pm\xi_\pm)}$ are the Weyl spinors which were expressed in terms of the elements t_{ij} of the matrix solution $T(\xi)$.

We conclude this section with the following important statement [19]. The boundary conditions for original variables $\boldsymbol{X}$ and Ψ are fulfilled, if the equality

$$\mathcal{M}_1(Q) = \epsilon 1 \tag{19}$$

† It is important that the resulting dependence from the ‘variable’ κ differs here from the dependence in [20].

holds for the monodromy matrix $\mathcal{M}_1$ of the system (13) defined in accordance with the equality $T(\xi + 2\pi) = T(\xi)\mathcal{M}_1$.

Thus the variables $\partial_\pm X_\mu$ and $\Psi_\pm$, constrained by the conditions (2) and (5), can be reconstructed through the matrix T—the matrix soluton of the linear 2π-periodic system (13). Moreover it is need that the coefficients Q_{ij} of this system are constrained by equality (19).

4. The topological charge and the definition of space $\mathcal{H}_1$

In this section we determine the adjunct phase space for the dynamical system (1). The starting point of our subsequent consideration is the correspondence (16). Note that the matrix solution $T(\xi)$ is defined up to within the transformations

$$T(\xi) \longrightarrow \tilde{T}(\xi) = T(\xi)B$$

where constant matrix $B \in SL(2, C)$. It is clear from the formulae (17) and (18) that these transformations are the Lorentz transformations of spacetime $E_{1,3}$. Thus we can write for every solution of the system (13):

$$T(\xi) = T_0(\xi)B_1(q_1, \ldots, q_6) \tag{20}$$

where the values q_i parametrize the group $SL(2, C)$ somehow or other and the matrix T_0 is defined from the functions $Q_{ij}(\xi)$ in some unique manner. In order to give the corresponding definition of the matrix T_0, let us fulfil the Iwasawa expansion for the matrix solution $T(\xi)$:

$$T = \mathcal{N}EU$$

where $\mathcal{U} \in \mathrm{SU}(2)$ and the matrices $\mathcal{E}$ and $\mathcal{N}$ are the following:

$$\mathcal{E} = \mathrm{diag}(e^d, e^{-d}) \qquad \mathcal{N} = \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix}.$$

After that we define the functions $j_a = j_a(\xi)$, $a = 1, 2, 3$:

$$j_a = -\mathrm{i}\,\mathrm{Tr}\,\sigma_a[\mathcal{G}^{-1}Q\mathcal{G} + \mathcal{G}^{-1}\mathcal{G}']$$

where $\mathcal{G} = \mathcal{N}\mathcal{E}$. Then the matrix $\mathcal{U}$ satisfies the following linear system:

$$\mathcal{U}' + \frac{\mathrm{i}}{2}\left(\sum_{a=1}^{3}\sigma_a j_a\right)\mathcal{U} = 0. \tag{21}$$

Because $\mathcal{U} \in \mathrm{SU}(2)$, the functions $j_a(\xi)$ are the real functions. It is more convienent to replace the function $\mathrm{d}(\xi)$, which defines the matrix $\mathcal{E}$, with the function $j_0(\xi) \equiv \mathrm{d}'(\xi)$ and the constant $d_0 = d(0)$.

We postulate the following six conditions to fix the matrix T_0:

$$\int_0^{2\pi} f(\xi)\,\mathrm{d}\xi = 0 \qquad d_0 = 0 \qquad \mathcal{U}(0) = \mathbf{1}.$$

Thus we define four real (j_a) and one complex (f) function such that the correspondence $Q \leftrightarrow (j_a; f)$ is one-to-one. Let us rewrite the condition (19) in terms of the introduced functions. So, the matrix $T_0(\xi)$ will be 2π-periodic if the functions $f(\xi)$, $j_0(\xi)$ are periodic and the equalities

$$\int_0^{2\pi} j_0(\xi)\,\mathrm{d}\xi = 0 \qquad \mathcal{U}(\xi + 2\pi) = \epsilon\mathcal{U}(\xi)$$

hold. The last equality means that the monodromy matrix $\mathcal{M}$ for the linear system (21) satisfies the condition

$$\mathcal{M} = \epsilon\mathbf{1}.$$

This constraint on the variables j_a leads to the appearance of the topological charge n in our model. Indeed, let us consider the spectral task

$$\mathcal{U}' + \frac{\mathrm{i}\lambda}{2}\left(\sum_{a=1}^{3}\sigma_a j_a\right)\mathcal{U} = 0.$$

Condition (21) holds if and only if this task has a point $\lambda_n = \lambda_n[j_a]$ of the periodic or antiperiodic spectrum such that $\lambda_n = 1$ for a certain number n. The equivalent form of this condition is the following:

$$\Phi_1^m \equiv \arccos(\tfrac{1}{2}\,\mathrm{Tr}\,\mathcal{M}) - \pi m = 0. \tag{22}$$

Thus we state the one-to-one correspondence

$$\boldsymbol{V}/E_{1,3} \approx (f(\xi), j_0(\xi), \ldots, j_3(\xi); q_1, \ldots, q_6; \kappa). \tag{23}$$

The whole number n is the topological charge in our theory; the continuous deformation of the string configuration $(f(\xi), \ldots)$ for some n into the configuration $(f(\xi), \ldots)$ with other number m breaks either boundary conditions or gauge (5).

Our next step is to define six parameters q_i according to the representation (20). Moreover, we must to add four constants Z_μ for the reconstruction of the variables X_μ from the derivatives $\partial_\pm X_\mu$. Let us consider the usual Noether expressions for the energy–momentum P_μ and the moment $M_{\mu\nu}$:

$$P_\mu = \frac{1}{4\pi\alpha'}\int_0^\pi \dot{X}_\mu\,\mathrm{d}\xi^1$$

$$M_{\mu\nu} = \frac{1}{4\pi\alpha'}\int_0^\pi (X_\mu\dot{X}_\nu - X_\nu\dot{X}_\mu)\,\mathrm{d}\xi^1 - \frac{\mathrm{i}\Theta}{8\pi\alpha'}\sum_{\epsilon=\pm}\int_0^\pi \overline{\Psi}_\epsilon(\Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu)\Psi_\epsilon\,\mathrm{d}\xi^1.$$

Let $w_\mu = -\frac{1}{2}\varepsilon_{\mu\nu\lambda\sigma}M^{\nu\lambda}P^\sigma$. In accordance with the formulae (17) and (18) we have the equalities:

$$(P)^2 = \left(\frac{\Theta}{4\pi\alpha'}\right)^2\sum_{l=0}^{2}\left(\frac{\kappa}{\Theta}\right)^l D_l \tag{24}$$

$$(w)^2 = \frac{\Theta^6}{(4\pi\alpha')^4}\sum_{l=0}^{6}\left(\frac{\kappa}{\Theta}\right)^l F_l. \tag{25}$$

It is important that the coefficients D_l and F_l in the polynomials (24) and (25) depend on the functions $f(\xi)$ and $j_a(\xi)$ only. This fact means that these formulae give the κ-parametric form of 'constraint'

$$\Phi_2(P^2, w^2; f, j_0, \ldots, j_3) = 0. \tag{26}$$

The main idea is to use the components P_μ and $M_{\mu\nu}$ as additional variables instead of the constants Z_μ, q_i and κ. The exact statement is as follows.

Corollary 1. *Let a two-parametric group G_2 be composed from the transformations:*

(1) rotations $X_\mu \to \Lambda_\mu^\nu(\phi)X_\nu$ in the space-like plane which is orthogonal with the vector P_μ and pseudo-vector w_μ;

(2) translations $X_\mu \to X_\mu + cP_\mu$.

Then, if the quantities $f(\xi)$, $j_a(\xi)$ ($a = 0, \ldots, 3$), P_μ and $M_{\mu\nu}$ are constrained by the equalities (22) and (26), the diffeomorphism

$$\boldsymbol{V}/G_2 \approx (f(\xi), j_0(\xi), \ldots, j_3(\xi); P_\mu, M_{\mu\nu})$$

takes place.

The sketch of the proof is as follows. Let the auxiliary vector field $\boldsymbol{X}_{(0)}$ and the spinor fields $\Psi_{(0)\pm}$ be defined from the variables $f(\xi)$ and $j_a(\xi)$ with the help of formulae (17) and (18), where the replacement $T(\xi) \to T_0(\xi)$ has been fulfilled. We next define the vector $P_{(0)\mu}$ and pseudo-vector $w_{(0)\mu}$ by means of the replacement $\boldsymbol{X} \to \boldsymbol{X}_{(0)}$ and $\Psi_\pm \to \Psi_{(0)\pm}$ in the correspondent Noether expressions. Let P_μ be an arbitrary time-like vector and $M_{\mu\nu}$ an arbitrary antisymmetrical tensor. Then, if the constraint (26) takes place, the matrix $B \in SL(2, C)$ exists such that the equalities

$$\hat{P} = B^+ \hat{P}_{(0)} B \qquad \hat{w} = B^+ \hat{w}_{(0)} B$$

hold. That is why we can reconstruct the matrix $T = T_0 B$. Moreover, we can restore the integration constants Z_μ because the full moment $M_{\mu\nu}$ consists of the information about the centre of mass of the string. Consequently, the original string variables $\boldsymbol{X}$ and $\Psi_\pm$ can be restored from the variables f, j_a, P_μ and $M_{\mu\nu}$. More detailed investigations show that this reconstruction will be smooth and that two-parametric arbitrariness exists, so that the corresponding cosets appear.

To describe the degrees of freedom connected with the group G_2, we introduce the additional coordinates q and θ, such that $-\infty < q < \infty$ and $\theta \in [0, 2\pi]$. Now we give the straightforward definition of the adjunct phase space $\mathcal{H}^{\mathrm{ad}}$ for the considered string model. This is a manifold such that any point $M \in \mathcal{H}^{\mathrm{ad}}$ has the following coordinates: (1) 2π-periodic complex function $f(\xi)$ without zero mode; (2) 2π-periodic real functions $j_a(\xi)$ (a=0,1,2,3) such that the function j_0 has not zero mode; (3) 4-vector P_μ such that the inequality $P^2 > 0$ holds; (4) antisymmetrical tenzor $M_{\mu\nu}$; (5) four additional coordinates q, θ, p and χ. Let us define the Poisson brackets

$$\{f(\xi), \overline{f}(\eta)\}^0 = \frac{\alpha'}{\Theta^2}\delta'(\xi - \eta) \qquad \{j_0(\xi), j_0(\eta)\}^0 = -2\frac{\alpha'}{\Theta^2}\delta'(\xi - \eta)$$

$$\{j_a(\xi), j_b(\eta)\}^0 = 2\frac{\alpha'}{\Theta^2}(-\delta_{ab}\delta'(\xi - \eta) + \varepsilon_{abc} j_c(\xi)\delta(\xi - \eta))$$

where $a, b, c = 1, 2, 3$, and $\delta(\xi) = \sum_n \mathrm{e}^{in\xi}$

$$\{M_{\alpha\beta}, M_{\gamma\delta}\}^0 = g_{\alpha\delta} M_{\beta\gamma} + g_{\beta\gamma} M_{\alpha\delta} - g_{\alpha\gamma} M_{\beta\delta} - g_{\beta\delta} M_{\alpha\gamma}$$
$$\{M_{\alpha\beta}, P_\gamma\}^0 = g_{\beta\gamma} P_\alpha - g_{\alpha\gamma} P_\beta$$
$$\{p, q\}^0 = 1 \qquad \{\chi, \theta\}^0 = 1.$$

(The other possible brackets are equal to zero.) With respect to the defined brackets the space $\mathcal{H}^{\mathrm{ad}}$ is the Poisson manifold.

The manifold $\boldsymbol{W}$ is defined as follows. First we require that the equalities

$$\Phi_3 \equiv p = 0 \qquad \Phi_4 \equiv \chi = 0$$

hold. As Φ_1^n we denote the constraint (22) for some topological number n. Let the set $\boldsymbol{W}_n \subset \mathcal{H}^{\mathrm{ad}}$ be the surface of the constraints $\Phi_i,\ i = 1, \dots, 4$, where $\Phi_2 = \Phi_2^n$. Then,

$$\boldsymbol{W} = \bigcup_{n\in Z} \boldsymbol{W}_n.$$

Corollary 2. *The constraints* $\Phi_i = 0, i = 1, \dots, 4$ *will be first-type constraints with respect to the brackets* $\{\cdot, \cdot\}^0$.

Indeed, $\{\Phi_i, \Phi_j\} = 0$ for $i = 1, 2$ and $j = 3, 4$, or $i = 3$ and $j = 4$. Let us prove that $\{\Phi_1, \Phi_2\} \propto \Phi_1$. We first note that the matrix $\mathcal{M}$ depends on the variables $j_a, a = 1, 2, 3$ only. Let us calculate the brackets of the matrix elements of the matrices $\mathcal{M}$ and $Q_g = (\mathrm{i}/2)\sum_a j_a\sigma_a$. The identity

$$\{(\mathcal{U}'(\xi) + Q_g(\xi)\mathcal{U}(\xi)) \otimes_, \mathcal{M}\}^0 \equiv 0$$

holds on space $\mathcal{H}^{\mathrm{ad}}$, therefore we can apply for such calculations the Leibniz rule $\{AB, C\}^0 = A\{B, C\}^0 + \{A, C\}^0 B$ and the definition of the matrix $\mathcal{M}$. As result we have the equality

$$\{Q_g(\xi) \overset{\otimes}{,} \mathcal{M}\}^0 = [1 \otimes \mathcal{M}, C(\xi)]$$

where the square brackets denote the commutator 4×4 matrices. The explicit form of the matrix $C(\xi)$ is not important here because it is clear that if $\mathcal{M} \propto 1$, then $\{Q_g(\xi) \overset{\otimes}{,} \mathcal{M}\}^0 \equiv 0$. Consequently, we have

$$\{\Phi_1, A\}^0 \propto \Phi_1$$

for arbitrary function $A = A(f, j_a; P_\mu, M_{\mu\nu}; q, \theta)$, so that the corollary is proven.

The dynamical equations

$$\{H_0, X_\mu\}^0 = \frac{\partial X_\mu}{\partial \xi^0} \qquad \{H_0, \Psi_\pm\}^0 = \frac{\partial \Psi_\pm}{\partial \xi^0}$$

hold for the Hamiltonian

$$H_0 = \frac{\Theta^2}{2\pi\alpha'}\left(\int_0^{2\pi} |f(\xi)|^2 \,\mathrm{d}\xi + \tfrac{1}{4}\sum_{a=0}^{3}\int_0^{2\pi} j_a^2(\xi)\,\mathrm{d}\xi\right).$$

These formulae can be proven with help of the representation (17) and (18) for original string variables X_μ, $\Psi_\pm$ and with help of the obvious equalities

$$\{H_0, j_a\}^0 = j_a' \qquad \{H_0, f\}^0 = f' \qquad \{H_0, P_\mu\}^0 = \{H_0, M_{\mu\nu}\}^0 = 0.$$

It can be verified directly that all constraints in our theory are coordinated with dynamics.

Remark. It is clear that the brackets of the variables P_μ and $M_{\mu\nu}$ are motivated by Poincaré algebra. Consequently, we have two annulators here: $P_\mu P^\mu$ and $w_\nu w^\nu$. But every Poisson structure $\{\cdot, \cdot\}$ must be coordinated with the tensor property of all considered functions. So, for instance, the equality $\{P_\mu, A_\nu\} = g_{\mu\nu}$ must hold for any 4-vector A_μ in order for the dynamical variables P_μ to generate Poincaré translations. In our theory the integration of formula (17) gives the expression for the radius vector $X_\mu(\xi^0, \xi^1)$:

$$\hat{X}(\xi^0, \xi^1) = \hat{Z} + \frac{\xi^0 + q}{\pi}\hat{P} - \frac{\mathrm{i}}{\pi}\sum_{n\neq 0}\frac{\hat{C}_n}{n}\mathrm{e}^{\mathrm{i}n\xi^0}\cos n\xi^1$$

where

$$\hat{C}_n = \int_0^{2\pi} T^\dagger(x)R(x)T(x)\mathrm{e}^{-\mathrm{i}nx}\,\mathrm{d}x$$

and $Z_\mu = M_{\mu\nu}P^\nu/P^2$. Therefore, we have the brackets

$$\{P_\mu, X_\nu\}^0 = g_{\mu\nu} - \frac{P_\mu P_\nu}{P^2}$$

which are coordinated with the fact that function P^2 will be an annulator. This means that for every constant 4-vector b^μ the following formula applies:

$$\mathrm{e}^{b^\mu\{P_\mu,\cdots}X_\nu = X_\nu + b_\nu - \left(\frac{b_\rho P^\rho}{P^2}\right)P_\nu.$$

Thus, with respect to the defined brackets, the variables P_μ will generate the Poincaré translations on the corresponding cosets only. The same situation holds for the rotations, mentioned in the definition of the group G_2. The additional constraints Φ_3 and Φ_4 allow us to reconstruct the correct coordination of the introduced Poisson brackets with translations and rotations. Indeed, let us consider the Lie operator

$$L^{\mu}(P) = \{P^{\mu}, \ldots + \pi \frac{P^{\mu}}{P^2}\{\Phi_3, \ldots$$

instead of the conventional operator of translation $\{P^{\mu}, \ldots$. In accordance with the definitions of the variable q and the constraint Φ_3, the equality

$$\mathrm{e}^{a_{\mu}L^{\mu}(P)}X_{\nu} = X_{\nu} + a_{\nu}$$

holds. Analogously, Lie operator $\{M^{\mu\nu}, \ldots$ must be improved by means of adding the term with the operator $\{\Phi_4, \ldots$.

Let us discuss the quantization of the suggested model. We surmise that the structure of the fundamental Poisson brackets algebra $\mathcal{A}_{\mathrm{cl}}$ gives some information about the constructed space of the quantum states. In our model this algebra has the form

$$\mathcal{A}_{\mathrm{cl}} = \mathcal{A}_{\mathrm{int}} \oplus \mathcal{P}$$

where $\mathcal{A}_{\mathrm{int}}$ the Poisson brackets algebra of the 'internal' variables $f(\xi)$, $j_a(\xi)$ and $\mathcal{P}$ is the Poincaré algebra. It should be emphasized that the energy–momentum and moment of the string (1) are independent fundamental variables, so there are no problems with the quantum ordering when we construct the quantum generators of Poincaré transformations.

The defined new variables are complicated functions from the original fields X and Ψ, which is why the correct introduction of quantum fermionic fields is not so obvious here. The following proposition clarifies this [20].

Corollary 3. *The equalities* $\Psi^{A}_{\pm}(\xi) \equiv$ *constant hold if and only if the equalities* $j_a(\xi) \equiv 0$ *for* $a = 0, \ldots, 3$ *take place.*

This statement means that, in spite of the complicated dependence of the variables f and j_a on the original variables X_{μ} and Ψ, the bosonic and fermionic degrees of freedom are still non-mixed. It is natural to fulfil the quantization of the variables j_a in terms of the fermionic fields with the help of the bosonization procedure [22]. Thus the natural Hilbert space of the of the quantum states of the string will be the following:

$$\boldsymbol{H} \bigoplus_{l,i,s} (\boldsymbol{H}_b \otimes \boldsymbol{H}_f \otimes \boldsymbol{H}_{\mu_i^2,s})$$

where the spaces $\boldsymbol{H}_{\mu^2,s}$ are the spaces of irreducible representations of Poincaré algebra $\mathcal{P}$, labelled by the eigenvalues of the Casimir operators $P^{\mu}P_{\mu}$ and $w^{\mu}w_{\nu}$; $\boldsymbol{H}_b$—the Fock space of two-dimensional bosonic field in the 'box' and $\boldsymbol{H}_f$—the Fock space of two-dimensional fermionic field in the 'box'. The corresponding physical vectors of states must be selected with the help of the 'Schrödinger equations'

$$\Phi_i|\psi_{\mathrm{phys}}\rangle = 0$$

where Φ_i are the quantum expressions for considered constraints.

The other consequence of this corollary is that the suggested theory can be considered as the new spinning generalization of the standard bosonic string model with the light-cone gauge. Indeed, the standard light-cone gauge for bosonic string can be written in the form

$$n_{\mu}\partial_{\pm}X^{\mu} = \pm p_{+}/2 \tag{27}$$

where light-like vector n_{μ} are selected usually as $(1, 0, 0, 1)$. In our case both spinors $\Psi_{\pm}$ are Majorana spinors in 4D spacetime, so the vectors $n^{\mu}_{\pm} = \overline{\Psi}_{\pm}\Gamma^{\mu}\Psi_{\pm}$ will be light-like always. The reduction $j_a \equiv 0$ means that these vectors are constant. Moreover $n^{\mu}_{+} = n^{\mu}_{-}$ in accordance with the usual boundary conditions for the spinor variables. Therefore, we have the theory with the gauge (27) where the light-like vector n_μ is constant, but arbitrary. If we require $\Theta = 0$, the action (1) takes the standard bosonic form. The real and imaginary parts of the functions $f(\pm\xi_\pm)$ will be the (well known) transversal components for vectors $\partial_\pm \boldsymbol{X}$. With respect to the formulae (5)

$$\Omega_{ij}\,\mathrm{d}\xi^i\,\mathrm{d}\xi^j \propto \kappa^2[(\mathrm{d}\xi_+)^2 + (\mathrm{d}\xi_-)^2]$$

so this form has a good limit when $\Theta \to 0$. In spite of this fact, we assume that the two-metric Ω_{ij} is not a natural object for the bosonic case $\Theta = 0$, because the spinor variables are absent here. This case was studied recently in [23], where both classical and quantum versions of the model were investigated in detail. As a result, we have Regge trajectories $\hbar\sqrt{s(s+1)} = \alpha_n\mu^2$, where the slopes α_n, $n = 1, 2, \ldots$ are the eigenvalues for some spectral task in the space of quantum states. The case $j_a \equiv 0$, but $\Theta \neq 0$ is quite similar technically, but is more interesting, because it leads to more complicated trajectories.

Note that we can fulfil some unusual reduction $f \equiv 0$ in our model which corresponds to the string, where all bosonic degrees of freedom are 'frozen'. Previous investigation of this case was made in [19], where the quantization was discussed too. This case is more complicated because of the (non-trivial) topological condition (22). The author hopes to study the general quantum case in the future.

5. Concluding remarks

In this paper we have suggested a new concept of adjunct phase space to investigate the open spinning string. It should be stressed that the suggested approach leads to 4D covariant theory both in the classical and in the quantum cases. The main result is a new non-trivial Regge spectrum which can be applied in our opinion to the description of exotic particles. The dependence $J = J(P^2)$, where the spin $J = \sqrt{w^2/P^2}$, can already be analysed on the classical level with the help of formulae (24) and (25). It will be essentially nonlinear for small masses although for large P^2 we have the asymptotics $J \propto P^2 + \mathcal{O}(\sqrt{P^2})$.

Let us note that we have two fundamental constants in the theory: α' and Θ. Because the spinor part of the action (1) vanishes on the equations of motion, the constant Θ can be introduced into the model not as a fixed constant but as an additional variable. Previous investigation of the theory with the original configuration space $(X, \Psi; \Theta)$ instead of the space (X, Ψ) was fulfilled in [19, 20]. It should be stressed that such extension leads to the scale-invariant theory if we define the scale transformations as $(X, \Psi; \Theta) \to (aX, \sqrt{a}\Psi; a\Theta)$. As a natural result, here the linear dependence $J \equiv \sqrt{s(s+1)} = \alpha_n\mu^2$ was deduced: we have the set of Regge trajectories with zero intercepts but with various slopes α_n. We considered the value Θ as constant not as variable in this work. Because the scale invariance is broken in this case the resulting Regge spectrum is more complicated than the spectrum in [19, 20]. Note that models of bosonic strings with non-standard spectrum were last suggested in [24, 25].

It is known that the slope α' of the Regge trajectory can be connected with the tension τ of the string: $\alpha' \propto 1/\tau$. If the tension is constant and there are no other internal forces, the slope will be constant too. Thus the complicated form of the Regge trajectory bears a relation, probably, to some additional internal forces within the string. Note that models where spinning degrees of freedom were connected with distributed charges and currents were investigated early on (see, for example, [26]). It appears that such an interpretation is possible in our case too. The interesting thing here is that the model has topological charge n which vanishes if the fermionic variables disappear.

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